

# SC07- Innovative Antenna Design: Insights Through Modal Decomposition



## Abstract

Electromagnetic wave phenomena, especially the guidance and radiation of electromagnetic waves, are commonly analyzed through modal representations, including normal and characteristic modes. This approach simplifies both analytical and numerical solutions, enabling effective analysis of complex engineering challenges. This course provides a comprehensive review of these techniques, covering traditional analytical methods and recent advances in numerical modeling. The instructors, with extensive careers dedicated to this field, will share their expertise, demonstrating a unified approach to modal decomposition techniques that facilitates practical problem-solving.

## Recommended prerequisites

Mathematical analysis and algebra (undergraduate level)  
Electromagnetic field theory (basic course)

## Learning objectives

1. Classify modal techniques.
2. Understand relevant mathematical tools (eigenvalue problems, separation of variables, orthogonality, spectral theorem, Rayleigh quotient, numerical range).
3. Implement and apply the techniques for solving given problems.
4. Apply characteristic mode theory, including the link between impedance and scattering formulations or relation to spherical waves.
5. Understand similarities between scattering formulation of characteristic modes (Garbacz's formulation) and impedance formulation (Harrington and Mautz), and the ways to unify them.
6. Use the properties of radiation modes to use them in practice.
7. Apply theoretical concepts in practical antenna design problem.

## Course outline

Part 1 (15 minutes): Introduction, modes (why to use them, motivation), and applications, overview and our classification

Part 2 (25 minutes): Mathematical background

- Orthogonality
- Rayleigh quotients
- Diagonalization
- Eigenvalue problems

Part 3 (40 minutes): Normal modes

- Separation of variables
- Diagonalization of lossless dynamical systems
- Waveguide and cavity modes with microwave applications
- Spherical vector waves and radiation patterns, model order reduction, near-field measurements
- Floquet modes
- Quasi-normal modes
- Number of propagating modes and Weyl's law
- Example: Degrees of freedom for communication systems

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## Course outline

Coffee break (15 min)

Part 4 (70 minutes): Characteristic modes and modes from diagonalization of input-output models

- Eigenvalues, orthogonality
- Example: Model-order reduction for scattering problems
- Equality of impedance-based modes and scattering modes
- Substructures (Schur complement + mention S, S0)
- CMA in an arbitrary basis
- Example: Feeding synthesis
- Example: Analysis of MIMO antennas

Part 5 (35 minutes): Diagonalization of optimization problems, radiation modes (45 min)

- Fundamental bounds on antenna performance
- Rayleigh quotients and quadratically constrained quadratic program (QCQP)
- Example: Radiation modes for radiation efficiency bound

Part 6 (10 minutes): Discussion and conclusion



**Mats Gustafsson** received the M.Sc. degree in Engineering Physics 1994, the Ph.D. degree in Electromagnetic Theory 2000, was appointed Docent 2005, and Professor of Electromagnetic Theory 2011, all from Lund University, Sweden. He co-founded the company Phase holographic imaging AB in 2004. His research interests are in scattering and antenna theory and inverse scattering and imaging. He has written over 100 peer-reviewed journal papers and over 100 conference papers. Prof. Gustafsson received the IEEE Schelkunoff Transactions Prize Paper Award 2010, IEEE Uslenghi Letters Prize Paper Award 2019, and Best Paper Awards at EuCAP 2007 and 2013. He served as an IEEE AP-S Distinguished Lecturer 2013-15.

**Miloslav Capek** received his Ph.D. degree in 2014, and was appointed Associate Professor in 2017, and Full Professor in 2023, all from the Czech Technical University in Prague, Czech Republic. Miloslav is a senior member of the IEEE, a former EurAAP delegate (Region 8, 2015-2020), and an Associate Editor of IET Microwaves, Antennas & Propagation. He has been a grant holder and member of a research team for national and international projects, including projects funded by the Czech Science Foundation (recipient of the Junior Star project), the Technology Agency of the Czech Republic, and the European Cooperation in Science and Technology. He leads the development of the AToM (Antenna Toolbox for MATLAB) package and serves as a vice-chair of the EurAAP "Software and Modeling" working group. He is the author or co-author of over 170 journal and conference papers. He received the IEEE Antennas and Propagation Edward E. Altshuler Prize Paper Award in 2022 and ESoA (European School of Antennas) Best Teacher Award in 2023. His current research interests include the areas of electromagnetic theory, electrically small antennas, numerical techniques, and optimization. For more detailed information, see [capek.elmag.org](http://capek.elmag.org)



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