

# SC11 - Fundamental equations of electromagnetics from classical to quantum: Theoretical formulation, efficient computation and enhanced learning

## Fields (insufficient)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \cdot \mathbf{D} = \rho$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0$$



## Potentials (gauge-dependent)

$$\nabla^2 \mathbf{A} - \mu\epsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}$$

$$\nabla^2 \phi - \mu\epsilon \frac{\partial^2 \phi}{\partial t^2} = -\frac{1}{\epsilon} \rho$$

## Abstract:

This short course will introduce new fundamental equations of electromagnetics (EM) that replace Maxwell's fields/potentials with single physical quantity unifying all electrostatics, magnetostatics, electrodynamics and quantum-EM interactions. Since Maxwell-Hertz-Heaviside era, the longstanding dilemma to use either fields or potentials and what gauge for electromagnetics will be discussed. The concept and utilization of field-impulses will then be shown to not only resolve such century-old field-potential/gauge dilemma, but also aptly describe quantum-EM (e.g. Aharonov-Bohm) effects. Theoretical formulation and efficient computation with fundamental implicit schemes of finite-difference-time-domain methods will be presented. Several mobile apps for technology-enhanced-learning of electromagnetics and transmission line circuits will also be demonstrated.

## Recommended pre-requisites:

The course requires some basic knowledge of:

- Electromagnetic theory, Maxwell's equations
- Fields and potentials in electromagnetics
- Finite-difference time-domain method

## Learning Objectives:

After the course the participant will be able to:

- Identify the equations in electromagnetics involving different fields and gauged potentials.
- Learn the pros and cons of using fields or potentials and gauges in electromagnetics.
- Understand the concept of field impulses and their utilization in electromagnetics.
- Formulate the fundamental equations of electromagnetics using field-impulses.
- Apply the finite-difference time-domain methods for computational electromagnetics.
- Understand the unconditionally stable fundamental implicit finite-difference time-domain schemes.
- Exploit the mobile apps for technology-enhanced learning of electromagnetics.

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## Course Outline:

The course outline is listed below:

### 1. Closer look into electromagnetic fields

We shall revisit Maxwell's formulations of his equations and some longstanding dilemmas and issues pertaining to either fields or potentials in electromagnetics.

### 2. Potentials and gauge

We shall discuss the significance of potentials comprising magnetic vector potential and electric scalar potentials.

### 3. Resolution using new fundamental physical quantity of electromagnetics

We shall provide satisfactory resolution to the longstanding dilemma of fields, potentials and gauge in lieu of the traditional fields and potentials.

### 4. Computation using FDTD

We shall briefly discuss the explicit FDTD for computation of fields with possible extension to field-impulses. We also present the unconditionally stable FDTD methods, particularly the efficient fundamental implicit schemes.

### 5. Technology-enhanced learning of EM

We shall address the challenges of teaching and learning of EM. We show how professors can make good use of the mobile apps for engaging interactive teaching of EM in class, and how students can utilize them for self-learning, checking of calculations, exploration etc.

## Instructor:



Eng Leong Tan is with the School of Electrical and Electronic Engineering (EEE), Nanyang Technological University (NTU), Singapore. He has published more than 140 papers and book chapters, most of which as a principal author. He has extensive research on electromagnetics and their extensions into multi-disciplinary physics including quantum, acoustics, thermal and circuits. He was the General Chair of Progress in Electromagnetics Research Symposium (PIERS) 2017 in Singapore. He was also the Technical Program Committee Chair for several key international conferences, including the 2021 IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting. He is a Fellow of the Electromagnetics Academy in recognition of his distinguished contributions to "Computational electromagnetics and education". He also received the undergraduate teaching award from IEEE Antennas and Propagation Society for excellence in teaching, student mentoring and the development of mobile technologies and computational methods for electromagnetics education. He has been appointed as the IEEE AP-S Distinguished Lecturer for 2025-2027 and MTT-S Speaker under TC-1 Field Theory and Computational EM Committee Speakers Bureau.

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