Trade-Offs in Multi-Element Receiving Antennas with Superconducting Feed Lines.

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Abstract—**Kinetic Inductance Detectors are widely accepted as receivers for Terahertz imaging. They are realized on superconductors and, exploiting the Cooper-pairs breaking by photons, they are able to sense incoming Terahertz radiation. In this paper we investigate the impact that the related absorption mechanism has on the angular dependence of the receiving system.**

I. INTRODUCTION

Terahertz technology has an important role in the acquisition of knowledge of some physical phenomena that are related to the origin of the universe and the formation of stars, galaxies and planets. The investigation of these phenomena, observable in the far-infrared range, requires very sensitive instrumentation for large imaging arrays. Superconducting detectors are a promising solution to this task [1].

This work is part of a collaboration between TNO (Netherlands Organization for Applied Scientific Research) and SRON (Netherlands Foundation for Space Research) regarding the investigation and development of technology for the future space mission SPICA [2]. The aim of SPICA is to observe the formation of galaxies and stars and to investigate the Cosmic Microwave Background (CMB). As part of the SPICA spacecraft the European instrument SAFARI has been proposed, for which three challenging requirements need to be met: a large number of pixels (of the order of 6000), a high sensitivity (NEP$\sim 10^{-19} W/\sqrt{\text{Hz}}$) and operation over a large frequency range (a decade divided in three different sub-bands, each of an octave). SRON has proposed Kinetic Inductance Detectors (KIDs) to satisfy the sensitivity requirement and they have involved TNO to design the integrated antenna to transfer the THz radiation to the KID and to design a large focal plane array.

A KID is a resonator that thanks to its superconducting properties can reach a very high quality factor (Q= 10^6). These properties allow the observation of very small changes in the resonance frequency, which are due to the Cooper-pairs breaking by means of the absorbed THz photons. These changes can be sensitively measured by a relatively simple microwave readout system.
In order to allow frequency multiplexing, an array of KIDs is needed. Each KID will be a resonator line with a slightly different length that will be coupled with the same CPW readout line. This length corresponds to a different resonance
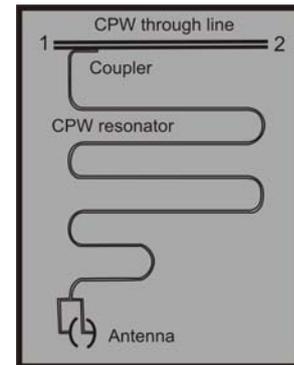

Fig. 1. Example of a KID coupled to an arc twin slot antenna [3]: the through line and the resonating quarter wavelength (at GHz frequencies) line are realized in CPW.

frequency. In this way thousands of pixels (the KIDs) can be placed on the same readout line. The very high sensitivity together with this simple readout system makes KIDs particularly attractive for very large focal plane array of SAFARI.

A KID needs to be coupled to an antenna in order to effectively absorb the incoming Terahertz radiation. Since the KID and the antenna are preferably on the same substrate and since the resonator is a CPW line, the most suitable antenna to couple to the detector is a slot antenna (Fig. 1).

For the antenna, we proposed to SRON two different designs: connected slots (Fig. 2) and a X-shaped slot (Fig. 3). The first design has a very large bandwidth and a high directivity [4], while the second has the advantage of a simple feed line with a single feed point.

Differences between simulations and measurements were observed in a preliminary 670 GHz demonstrator [5]. These differences suggest that the Cooper-pairs breaking, related to the photon absorption mechanism, causes the CPW lines at 670 GHz to be very lossy. In this work, we investigate the effect of this particular absorption mechanism on the antenna-array performance. The aim is to formulate a criterion to select the antenna design that best fits this specific application.

A multiple-feed antenna exhibits a large gain if the contributions received by each array element add up coherently. However the "loss" related to the Cooper-pairs breaking could
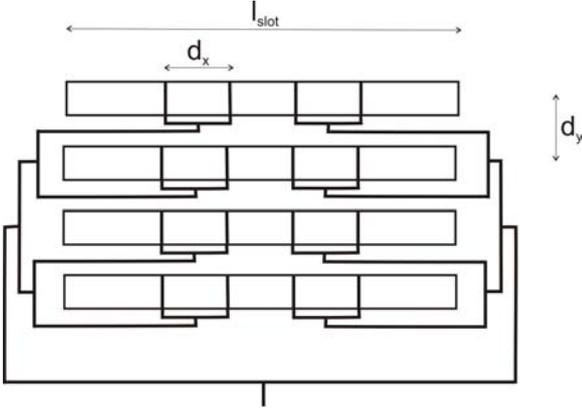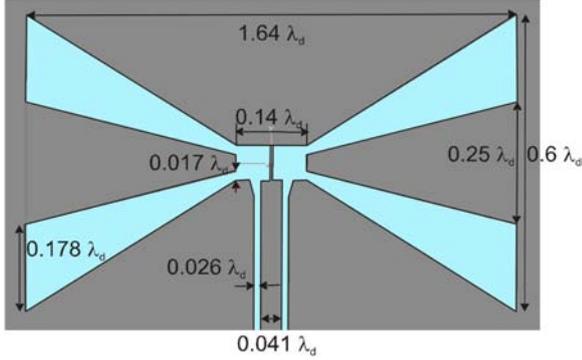
Fig. 2. Array of connected slot 4x4 and feeding line.

Fig. 3. X shaped slot antenna.

$$I_3 = g(\theta)\left(\frac{I_0}{\sqrt{2}}e^{jk_0\frac{d}{2}\cos\theta} + \frac{I_0}{\sqrt{2}}e^{-jk_0\frac{d}{2}\cos\theta}\right)e^{-jk_l l} =$$

$$= g(\theta)\frac{2I_0}{\sqrt{2}}\cos(k_0 d/2\cos\theta)e^{-\alpha l}e^{-j\beta l}.$$

In case the lines are lossless ($\alpha = 0$), the entire power is received according to the pattern $g^2(\theta)\cos^2(k_0 d/2\cos\theta)$. However, for frequencies above the gap frequency, the superconductor behaves as a normal metal and $\alpha \neq 0$. An expression for the attenuation constant of CPW is given in [6]. In case the lines are lossy ($\alpha \neq 0$), $e^{-2\alpha l}$ of the power is received coherently according to the pattern $g^2(\theta)\cos^2(k_0 d/2\cos\theta)$, while $(1 - e^{-2\alpha l})$ is received incoherently according to the square of the element pattern. These two contributions will be indicated with $P_c(\theta)$ (coherent) and $P_i(\theta)$ (incoherent) respectively. For lines etched on superconducting metals both these contributions will affect the behavior of the KID and therefore they should be taken into account in the evaluation of the radiation pattern. For $I_0 = 1$ A, we find

$$P(\theta) = P_c(\theta) + P_i(\theta) =$$

$$= (1 - e^{-2\alpha l})g^2(\theta) +$$

$$+ 2e^{-2\alpha l}\left(g(\theta)\cos(k_0\frac{d}{2}\cos\theta)\right)^2. \qquad (1)$$

We can now generalize this formula for the case of a uniform linear array of N elements fed by a corporate feeding network. All the branches of this network have the same characteristic impedance Z_0. We consider the array radiating in a medium with propagation constant k. The branches of
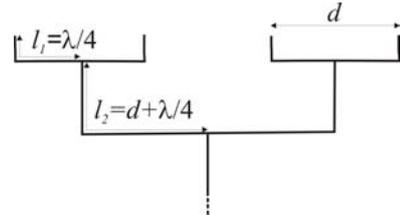

Fig. 4. Example of a linear array with 4 elements.

the feeding line are composed of different pieces $l_1, l_2, ..., l_M$ with $M = \log_2 N$, where N is the number of elements and M is the number of power combination levels (Fig. 4). The total current flowing in the final output line is

$$I = \frac{I_0}{(\sqrt{2})^M}g(\theta)\left|\frac{e^{(jNkd\cos\theta)} - 1}{e^{(jkd\cos\theta)} - 1}\right| \times$$

$$\times \quad e^{-\alpha(l_1+...+l_M)}e^{-j\beta(l_1+...+l_M)}. \qquad (2)$$

be such that a larger number of elements in the array does not increase the performance, since the contributions from the physically separated antennas do not add up entirely in a coherent way.

In this paper, we present a trade off study that leads to the determination of a maximum number of pixels, depending on the expected losses.

II. Coherent and Incoherent contributions

First we consider an array composed of two elements, each fed by a transmission line, characterized by an element pattern $g(\theta)$. The power incident on the elements is equally divided over the two branches of the feeding line. The two lines have length l and are connected in parallel. For our analysis we assume that the single resulting line is infinitely extended.

Our aim is to evaluate the angular dependence of the power received by the system and actually used to break the Cooper-pairs.

The current at each antenna element can be expressed as $I_1 = I_0 e^{jk_0 d/2\cos\theta}$ and $I_2 = I_0 e^{-jk_0 d/2\cos\theta}$, where $\theta \in [0, \pi]$ is the angle of incidence, I_0 is a constant amplitude factor, k_0 is the propagation constant in vacuum and d is the spacing between the antenna elements. The two transmission lines are of equal length and impedance. At the combiner, the power is summed up resulting in the current I_3 given by

Using 2, we can write the coherent contribution of the power as

$$P_c(\theta) = Z_0 \frac{I_0^2}{(\sqrt{2})^{2M}} g^2(\theta) \left| \frac{e^{(jNkd\cos\theta)} - 1}{e^{(jkd\cos\theta)} - 1} \right|^2 \times$$

$$\times \quad e^{-2\alpha(l_1+\ldots+l_M)}. \tag{3}$$

The final output line from the common feeding point is infinitely extended and therefore, if lossy, all the power provided coherently by the antenna will be absorbed along this line. Complete absorption is a good approximation also in the actual application, where the line corresponds to the KID. Since the resonator line of the KID is very long at THz frequencies, all the power can be considered absorbed along this line.

In the general case of a linear array with N elements, the total received power is

$$P(\theta) = P_c(\theta) + P_{i1}(\theta)N + P_{i2}(\theta)\frac{N}{2} + \ldots +$$

$$+ \quad P_{iM}(\theta)\frac{N}{2^{M-1}} \tag{4}$$

where the factor $N/2^{m-1}$, $m = 1,\ldots,M$, is the number of branches for each power combination level and $P_{i1}(\theta), P_{i2}(\theta), \ldots, P_{iM}(\theta)$ are the incoherent contributions at each of these levels:

$$P_{i1}(\theta) = \frac{1}{2}Z_0 I_0^2 g^2(\theta) \cdot (1 - e^{-2\alpha l_1}) \cdot 2$$

$$P_{i2}(\theta) = \frac{1}{2}Z_0 I_0^2 \left(\sqrt{2}g(\theta) \cos(k_0 \frac{d}{2}\cos\theta) \right)^2 \times$$

$$\times \quad e^{-2\alpha l_1}(1 - e^{-2\alpha l_2}) \cdot 2$$

$$\vdots$$

$$P_{iM}(\theta) = \frac{1}{2}Z_0 I_0^2 \left((\sqrt{2})^{M-1} g(\theta) \cos(k_0 \frac{d}{2}\cos\theta) \right)^2 \times$$

$$\times \quad \left(\prod_{p=1}^{M} \cos[k_0(p-1)d\cos\theta] \right)^2 \times$$

$$\times \quad e^{-2\alpha l_1} \cdot \ldots \cdot e^{-2\alpha l_{M-1}}(1 - e^{-2\alpha l_M}) \cdot 2. \tag{5}$$

Here, the additional factor 2 is due to the fact that a sub-array consists of two elements. The idea is that each sub-array of two elements is contributing to the incoherent power with a different weight depending on its own element factor.

III. RESULTS

The previous section has described the expression of the absorbed power in a multi-element feed network highlighting the influence of the attenuation constant on the angular sensitivity of such antenna system. In this section, results are presented for an array with a constant element factor and with a different number of elements (2, 4, 8 and 16). The frequency is 1 THz. The spacing of the array is $d = 0.5\lambda_d$, where λ_d is the wavelength in the medium where the radiation comes from. We suppose that the array has a Silicon lens glued on it ($\varepsilon_r = 11.9$). Fig. 4 shows an example of a feeding line for a linear array with four elements. In figures 5, 6, 7 and 8, the angular sensitivity $P(\theta)$ is plotted versus the angle θ for the lossless case and for the case of different values of the attenuation constant, where arrays with 2, 4, 8 and 16 elements are considered respectively.
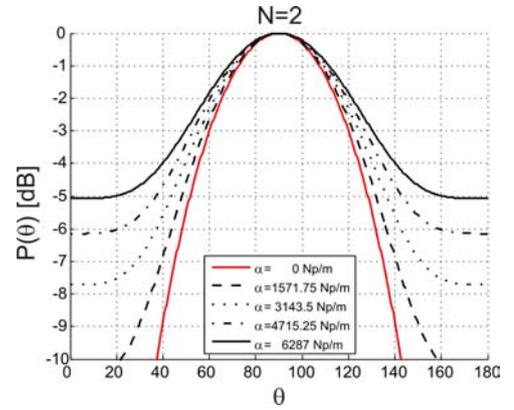

Fig. 5. Angular sensitivity of an array with 2 of elements with constant element factor for different values of attenuation constant.
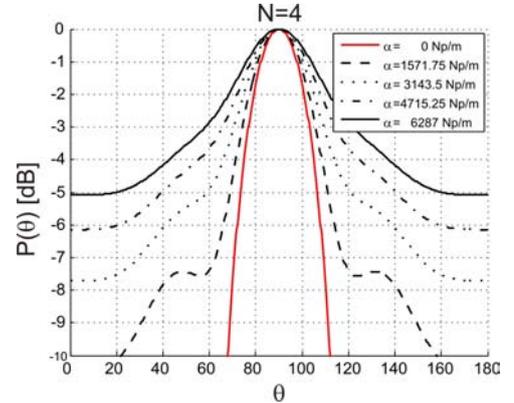

Fig. 6. Angular sensitivity of an array with 4 of elements with constant element factor for different values of attenuation constant.

We observe that near end-fire (θ=0, θ=180) the sensitivity is constant for each array size and each choice of the attenuation constant. The level of this floor is dictated by the power absorbed in the first branch of the feeding line. We note that in our graphs the floor does not depend on the number of elements of the array because of normalization. We consider
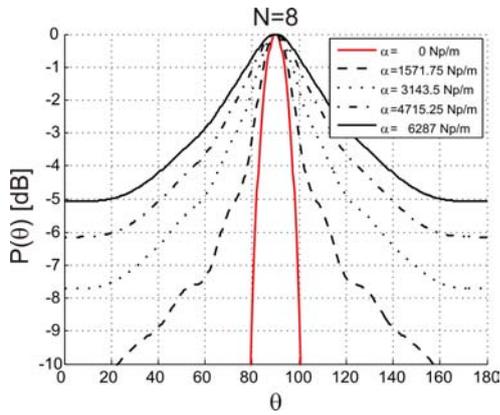
Fig. 7. Angular sensitivity of an array with 8 of elements with constant element factor for different values of attenuation constant.
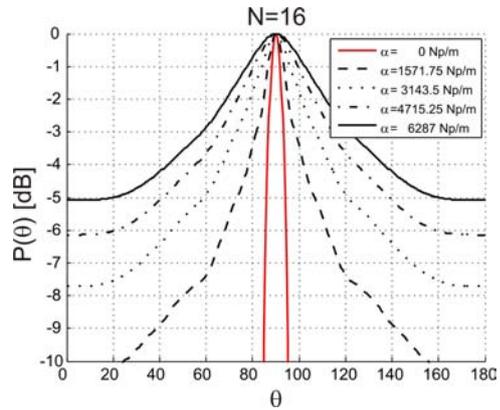

Fig. 8. Angular sensitivity of an array with 16 of elements with constant element factor for different values of attenuation constant.

the floor as reference to establish the level of performance degradation of our system. If the floor is above the threshold of -10 dB, we consider the behavior of our system not reliable because of the high losses. As can be observed from the figures, for values of attenuation approximately above 1500 Np/m, the angular sensitivity at end-fire is above -10 dB. Also the level of the sidelobes increases, which strongly affects the antenna behavior.

Our model is represented schematically in Fig. 9. The phase distribution of an incident plane-wave is generated by the upper part of the circuit of this figure. The source power flowing from port 1 is equally divided by an ideal power divider into two ideal transmission lines with phase difference $kd\cos\theta$. Each of these lines is further connected to a wavelength long transmission line that is lossy. The feed network consists of an ideal power combiner (we assume no coupling between the combined branches and no reflections at the junction).

The S_{21} parameter of the equivalent circuit is a measure for the coherent received power. We need also to sum up the incoherent power contributions to obtain the angular sensitivity. The effect of using actual components can be taken into account by replacing the ideal power combiner with a device
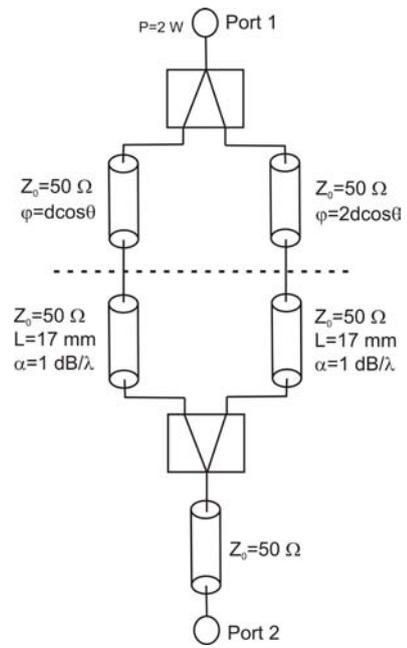

Fig. 9. Equivalent circuit representing a plane wave impinging on an array of two elements at an angle θ from the array axis.

whose scattering parameters are obtained by using a full wave simulator. Both reflected and coupled powers will be absorbed in the two lossy lines, while a quantity of power, which is smaller than the one predicted by the ideal case, will flow in the line below the common point.

IV. CONCLUSIONS

Kinetic Inductance Detectors are very promising devices for THz detection. They are realized on superconducting metals and are coupled with antennas to more effectively sense the incoming THz signal. In this paper we showed the effect that absorption in the feed network has on the angular sensitivity of arrays realized on superconducting materials. Our study gives a criterion for the number of elements an array requires to obtain a specified performance, depending on the attenuation constant of the transmission lines. For attenuation constants up to approximately 1500 Np/m the pattern degrades negligibly (at least for arrays up to 16 elements), because the end-fire sensitivity is below -10 dB. A trade-off study is to be found depending on the objectives of the investigation and the pattern required by the receiver.

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